Simple and Efficient Finite-Element Analysis of Microwave and Optical Waveguides

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***Abstract*—A simple and efficient finite-element method for the analysis of microwave and optical waveguiding problems is formulated using three components of the electric or magnetic field. In order to eliminate spurious solutions, edge elements are introduced. In the edge element approach the nodal parameters are not limited to the magnetic field as in the conventional three-component formulation for the dielectric waveguiding problem. An eigenvalue equation derived here involves only the edge variables in the transversal plane and can provide a direct solution for the propagation constant. To show the vailidity and usefulness of this approach, computed results are illustrated for microstrip transmission lines and dielectric waveguides.**

I. Introduction

To rigorously evaluate propagation characteristics of microwave and optical waveguides with arbitrarily shaped cross sections, vectorial wave analysis is necessary, and different types of the vector finite-element method (FEM) have been developed. Of the various formulations, the FEM using full vector $\boldsymbol{H}$ field is quite suitable for a wide range of practical, complicated problems [1]–[10]. This approach has been widely used for various dielectric waveguiding structures in microwave, millimeter-wave, and optical wavelength regions, and recently has been utilized as the waveguide solver of CAD packages [7]. The most serious problem associated with this approach is the appearance of spurious solutions. The penalty function method [3], [4], [6], [7] has been used to cure this problem, but in this technique an arbitrary positive constant, called the penalty coefficient, is involved and the accuracy of solutions depends on its magnitude. Furthermore, in the full vectorial formulation [1]–[10] the propagation constant is first given as an input datum, and subsequently the operating frequency is obtained as a solution. More recently, several methods for solving directly the propagation constant have been developed, but each has its drawback, e.g., a large number of field components [11]–[13], consideration of the adjoint field which does not correspond to the actual electromagnetic field [14], or the need to estimate the line integral in the variational expression [15].

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In this paper a simple and efficient FEM for the analysis of microwave and optical waveguiding problems is formulated using three components of the electric or magnetic field. In order to eliminate spurious solutions and to treat arbitrarily shaped waveguides, triangular edge elements are introduced. An eigenvalue equation derived here involves only the edge variables in the transversal plane and can provide a direct solution for the propagation constant. To show the validity and usefulness of this approach, examples are computed for microstrip transmission lines on isotropic or anisotropic substrates, dielectric rectangular waveguides, and equilateral triangular core waveguides.

II. Basic Equations

We consider a dielectric waveguide with a diagonal permittivity tensor and assume that the electromagnetic field in the waveguide varies as $\exp[j(\omega t - \beta z)]$, where t is the time, z is the propagation direction, ω is the angular frequency, and β is the propagation constant in the z direction.

From Maxwell's equations the following vectorial wave equation is derived:

$$\nabla \times ([p]\nabla \times \phi) - k_0^2[q]\phi = 0 \tag{1}$$

with

$$[p] = \begin{bmatrix} p_x & 0 & 0 \\ 0 & p_y & 0 \\ 0 & 0 & p_z \end{bmatrix} \tag{2}$$

$$[q] = \begin{bmatrix} q_x & 0 & 0 \\ 0 & q_y & 0 \\ 0 & 0 & q_z \end{bmatrix} \tag{3}$$

where k_0 is the free-space wavenumber, ϕ denotes either $\boldsymbol{E}$ or $\boldsymbol{H}$, and the components of $[p]$ and $[q]$ are given by

$$\begin{aligned} p_x &= p_y = p_z = 1, \\ q_x &= \epsilon_{rx} = n_x^2, \\ q_y &= \epsilon_{ry} = n_y^2, \\ q_z &= \epsilon_{rz} = n_z^2 \qquad \text{for } \phi = \boldsymbol{E} \end{aligned} \tag{4}$$

$$p_x = 1/\epsilon_{rx} = 1/n_x^2,$$
$$p_y = 1/\epsilon_{ry} = 1/n_y^2,$$
$$p_z = 1/\epsilon_{rz} = 1/n_z^2,$$
$$q_x = q_y = q_z = 1 \quad \text{for } \phi = \boldsymbol{H}. \tag{5}$$

Here ϵ_{rx}, ϵ_{ry}, ϵ_{rz} are the relative permittivities in the x, y, z directions, respectively, and n_x, n_y, n_z are the refractive indices in the x, y, z directions, respectively.

The functional for (1) is given by

$$F = \iint_\Omega [(\nabla \times \phi)^* \cdot ([p]\nabla \times \phi) - k_0^2[q]\phi^* \cdot \phi]\, dx\, dy \tag{6}$$

where Ω is the waveguide cross section and the asterisk denotes complex conjugate.

III. Finite Element Formulation

The electromagnetic fields have to be tangentially continuous across material interfaces. In the edge element [5], [8]–[10], [16]–[20], the tangential continuity can be straightforwardly imposed. Hano [5] has developed the FEM with rectangular edge elements for solving inhomogeneous waveguiding problems. Kikuchi [16], on the other hand, has utilized triangular edge elements to treat arbitrarily shaped waveguides, but in [16] only the homogeneous hollow waveguides are analyzed.

In this section we apply the triangular edge element [16], which is different from that used by Hano [8], to inhomogeneous waveguiding problems.

A. Triangular Edge Element

The six nodes described in the triangular edge element consist of the three corner and three side points as shown in Fig. 1. The corner points 1 to 3 are for the axial component ϕ_z (E_z or H_z), while the side points 4 to 6 are for the tangential component ϕ_t (E_t or H_t).

The axial component ϕ_z is approximated by a complete polynomial of first order:

$$\phi_z = j\{N(x, y)\}^T\{\phi_z\}_e = j\{N\}^T\{\phi_z\}_e \tag{7}$$

with

$$\{N\} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \frac{1}{2A_e}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \tag{8}$$

where $\{\phi_z\}_e$ is the nodal axial-field vector for each element, $\{N\}$ is the ordinary shape function vector for the linear triangular element, L_k's ($k = 1, 2, 3$) are the area coordinates, and the area of the element, A_e, and the coefficients a_k, b_k, c_k are given by

$$2A_e = \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \tag{9}$$

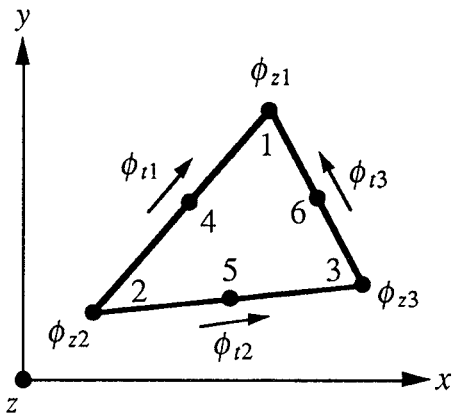

Fig. 1. Triangular edge element.

$$a_k = x_l y_m - x_m y_l \tag{10}$$
$$b_k = y_l - y_m \tag{11}$$
$$c_k = x_m - x_l. \tag{12}$$

Here x_k, y_k ($k = 1, 2, 3$) are the Cartesian coordinates of the corner points 1 to 3 of the triangle and the subscripts k, l, m always progress modulo 3, i.e., cyclically around the three vertices of the triangle.

The transverse components ϕ_x (E_x or H_x) and ϕ_y (E_y or H_y) are approximated by a linear function of y and x, respectively:

$$\phi_x = \{U(y)\}^T\{\phi_t\}_e = \{U\}^T\{\phi_t\}_e \tag{13}$$
$$\phi_y = \{V(x)\}^T\{\phi_t\}_e = \{V\}^T\{\phi_t\}_e \tag{14}$$

with

$$\{U\} = \begin{bmatrix} \tilde{a}_1 + \tilde{c}_1 y \\ \tilde{a}_2 + \tilde{c}_2 y \\ \tilde{a}_3 + \tilde{c}_3 y \end{bmatrix} \tag{15}$$

$$\{V\} = \begin{bmatrix} \tilde{b}_1 - \tilde{c}_1 x \\ \tilde{b}_2 - \tilde{c}_2 x \\ \tilde{b}_3 - \tilde{c}_3 x \end{bmatrix} \tag{16}$$

where $\{\phi_t\}_e$ is the edge variables in the transversal plane for each element. $\{U\}$ and $\{V\}$ are the shape function vectors for the triangular edge element, and the coefficients $\tilde{a}_k$, $\tilde{b}_k$, $\tilde{c}_k$ are given by

$$\tilde{a}_k = [(y_{m+3}\cos\theta_{m+3} - x_{m+3}\sin\theta_{m+3})\sin\theta_{l+3} - (y_{l+3}\cos\theta_{l+3} - x_{l+3}\sin\theta_{l+3})\sin\theta_{m+3}]/\Delta \tag{17}$$

$$\tilde{b}_k = [(y_{l+3}\cos\theta_{l+3} - x_{l+3}\sin\theta_{l+3})\cos\theta_{m+3} - (y_{m+3}\cos\theta_{m+3} - x_{m+3}\sin\theta_{m+3})\cos\theta_{l+3}]/\Delta \tag{18}$$

$$\tilde{c}_k = (\cos\theta_{l+3}\sin\theta_{m+3} - \cos\theta_{m+3}\sin\theta_{l+3})/\Delta \tag{19}$$

with

$$0 \le \theta_{k+3} = \tan^{-1}\{(y_k - y_l)/(x_k - x_l)\} < \pi \tag{20}$$

$$\Delta = \sum_{k=1}^{3} (y_{k+3}\cos\theta_{k+3} - x_{k+3}\sin\theta_{k+3}) \cdot (\cos\theta_{l+3}\sin\theta_{m+3} - \cos\theta_{m+3}\sin\theta_{l+3}). \tag{21}$$

Here x_{k+3}, y_{k+3} ($k = 1, 2, 3$) are the Cartesian coordinates of the side points 4 to 6 of the triangle.

Note that the tangential component, $\phi_t = \phi_x \cos\theta + \phi_y \sin\theta$, is continuous along the interelement boundaries and is constant on each side of triangles.

B. Finite-Element Discretization

Dividing the waveguide cross section into a number of edge elements, we expand the transverse components ϕ_x, ϕ_y and the axial component ϕ_z in each element as

$$\phi = [N]^T\{\phi\}_e \tag{22}$$

with

$$\{\phi\}_e = \begin{bmatrix} \{\phi_t\}_e \\ \{\phi_z\}_e \end{bmatrix} \tag{23}$$

$$[N] = \begin{bmatrix} \{U\} & \{V\} & \{0\} \\ \{0\} & \{0\} & j\{N\} \end{bmatrix} \tag{24}$$

where $\{0\}$ is a null vector.

Substituting (22) into (6), from the variational principle we obtain the following eigenvalue problem:

$$[K]\{\phi\} - k_0^2[M]\{\phi\} = \{0\} \tag{25}$$

with

$$[K] = \begin{bmatrix} [K_{tt}] & [K_{tz}] \\ [K_{zt}] & [K_{zz}] \end{bmatrix} = \sum_e \iint_e [B] * [p][B]^T\, dx\, dy \tag{26}$$

$$[M] = \begin{bmatrix} [M_{tt}] & [0] \\ [0] & [M_{zz}] \end{bmatrix} = \sum_e \iint_e [N] * [q][N]^T\, dx\, dy \tag{27}$$

$$[B] = \begin{bmatrix} j\beta\{V\} & -j\beta\{U\} & -\{U_y\} + \{V_x\} \\ j\{N_y\} & -j\{N_x\} & \{0\} \end{bmatrix} \tag{28}$$

where $\{\phi\}$ is the global field vector and the submatrices of $[K]$ and $[M]$ are given by

$$[K_{tt}] = \sum_e \iint_e [p_x\beta^2\{V\}\{V\}^T + p_y\beta^2\{U\}\{U\}^T + 4p_z\{U_y\}\{U_y\}^T]\, dx\, dy \tag{29a}$$

$$[K_{tz}] = [K_{zt}]^T = \sum_e \iint_e [p_x\beta\{V\}\{N_y\}^T + p_y\beta\{U\}\{N_x\}^T]\, dx\, dy \tag{29b}$$

$$[K_{zz}] = \sum_e \iint_e [p_x\{N_y\}\{N_y\}^T + p_y\{N_x\}\{N_x\}^T]\, dx\, dy \tag{29c}$$

$$[M_{tt}] = \sum_e \iint_e [q_x\{U\}\{U\}^T + q_y\{V\}\{V\}^T]\, dx\, dy \tag{30a}$$

$$[M_{zz}] = \sum_e \iint_e q_z\{N\}\{N\}^T\, dx\, dy. \tag{30b}$$

Here $\{N_x\} \equiv \partial\{N\}/\partial x$, $\{N_y\} \equiv \partial\{N\}/\partial y$, $\{U_y\} \equiv d\{U\}/dy$, and $\{V_x\} \equiv d\{V\}/dx$.

Equation (25) may be rewritten as

$$[K_{tt}]\{\phi_t\} - \beta[K_{tz}]\{\phi_z\} - \beta^2[M_{tt}]\{\phi_t\} = \{0\} \tag{31a}$$

$$-\beta[K_{zt}]\{\phi_t\} + [K_{zz}]\{\phi_z\} = \{0\} \tag{31b}$$

with

$$[K_{tt}] = \sum_e \iint_e [q_x k_0^2\{U\}\{U\}^T + q_y k_0^2\{V\}\{V\}^T - 4p_z\{U_y\}\{U_y\}^T]\, dx\, dy \tag{32a}$$

$$[K_{tz}] = [K_{zt}]^T = \sum_e \iint_e [p_x\{V\}\{N_y\}^T + p_y\{U\}\{N_x\}^T]\, dx\, dy \tag{32b}$$

$$[K_{zz}] = \sum_e \iint_e [q_z k_0^2\{N\}\{N\}^T - p_x\{N_y\}\{N_y\}^T - p_y\{N_x\}\{N_x\}^T]\, dx\, dy \tag{32c}$$

$$[M_{tt}] = \sum_e \iint_e [p_x\{V\}\{V\}^T + p_y\{U\}\{U\}^T]\, dx\, dy. \tag{33}$$

Note that the submatrices in (32) and (33) are different from those in (29) and (30).

Substituting (31b) into (31a), we obtain the following final eigenvalue problem:

$$[\tilde{K}_{tt}]\{\phi_t\} - \beta^2[\tilde{M}_{tt}]\{\phi_t\} = \{0\} \tag{34}$$

with

$$[\tilde{M}_{tt}] = [M_{tt}] + [K_{tz}][K_{zz}]^{-1}[K_{zt}]. \tag{35}$$

Note that (34) will give a solution directly for the propagation constant and the corresponding field distribution, and involves only the edge variables in the transversal plane $\{\phi_t\}$. But it is important to point out the price paid for this: a matrix inversion has to be performed and the sparsity of the matrices is destroyed.

The integrals necessary to construct element matrices are summarized in the Appendix.

IV. Numerical Examples

First, we consider a microstrip transmission line in Fig. 2 and subdivide one-half of the waveguide cross section into edge elements, where $W = 1.27$ mm, $t = 0$, $h = 1.27$ mm, $X = 12.7$ mm, and $Y = 12.7$ mm. Fig. 3(a) and (b) show the propagation characteristics for the first two modes of a microstrip on an isotropic substrate with $\epsilon_r = 8.875$ and for those of a microstrip on an anisotropic substrate with $\epsilon_{rx} = \epsilon_{rz} = 9.4$ and $\epsilon_{ry} = 11.6$, respectively, where the number of elements $N_E = 364$, the number of corner points $N_C = 210$, and the number of side points $N_S = 573$. Our results agree well with previously reported ones for both isotropic [21]–[23] and anisotropic [23], [24] cases.

Next, we consider a dielectric rectangular waveguide in Fig. 4, where n_1 and n_2 are the refractive indices of the core and cladding regions, respectively. Because of the twofold symmetry of the system, we subdivide only one-quarter of the waveguide cross section into edge elements. For simplicity, assuming the artificial boundaries $x = \pm X/2$ and $y = \pm Y/2$ far from the core region, the original unbounded structure is replaced by a corresponding bounded one. Here, the conditions for the perfect electric or perfect magnetic conductors are imposed suitably on the artificial boundaries, so as not to restrict the dominant electromagnetic field component there. Fig. 5 shows the propagation characteristics of this waveguide, where $W = 2t$, $X = 10t$, $Y = 5t$, $N_E = 320$, $N_C = 187$, $N_S = 506$, and the normalized frequency v and the normalized propagation constant b are defined as

$$v = k_0 t \sqrt{n_1^2 - n_2^2}/\pi \tag{36}$$

$$b = \frac{(\beta/k_0)^2 - n_2^2}{n_1^2 - n_2^2}. \tag{37}$$

Our results agree well with the results of the point matching method [25]. The results of the Marcatili's method [26] deviate from those of the point matching method at lower frequencies.

Lastly, we consider an equilateral triangular core waveguide in Fig. 6 and subdivide one-half of the waveguide cross section into edge elements. Fig. 7 shows the propagation characteristics for the E_{11}^y mode of this waveguide, where $X = 6t$, $Y = 5t$, $N_E = 360$, $N_C = 208$, and $N_S = 567$. The finite-element solutions of edge element formulation agree well with those of axial-field (E_z and H_z) formulation [27] and those of full vector H-field formulation with the penalty coefficient $s = 1$ [28].

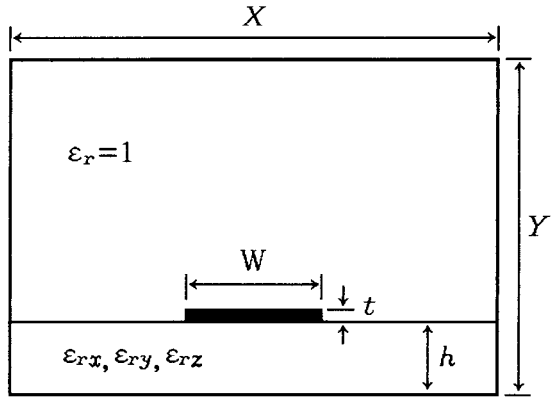

Fig. 2. Shielded microstrip transmission line.

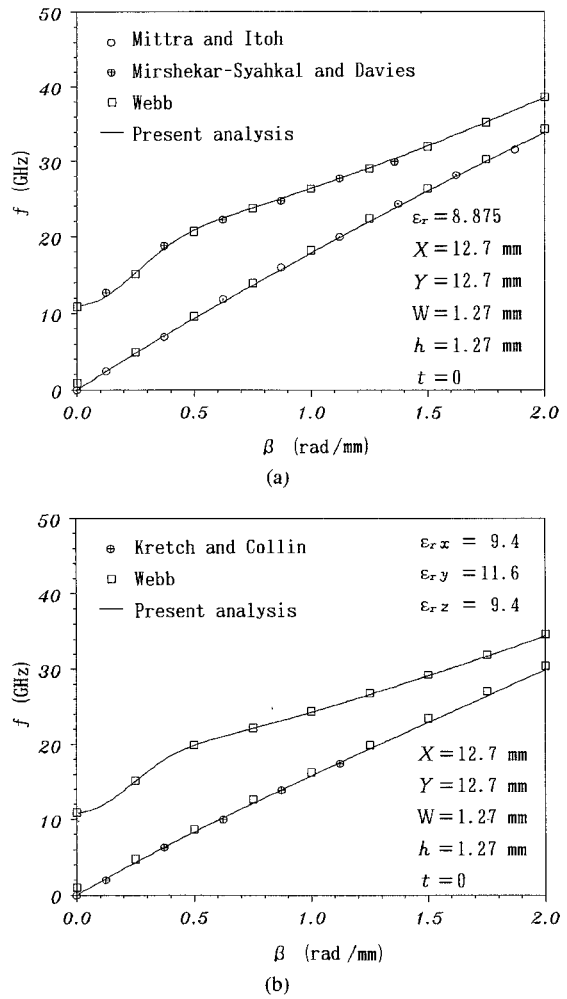

Fig. 3. Propagation characteristics of a microstrip transmission line. (a) Isotropic substrate. (b) Anisotropic substrate.

Note that the spurious solutions are included in the finite-element solutions of axial-field formulation. To avoid confusion, such spurious solutions are not shown in Fig. 7(a). In the edge element method spurious solutions do not appear anywhere. Furthermore, the newly derived eigenvalue problem (34) does not produce zero eigenvalues [5], [8]–[10] which are present in (25). The convergence of solutions has been checked by increasing the number

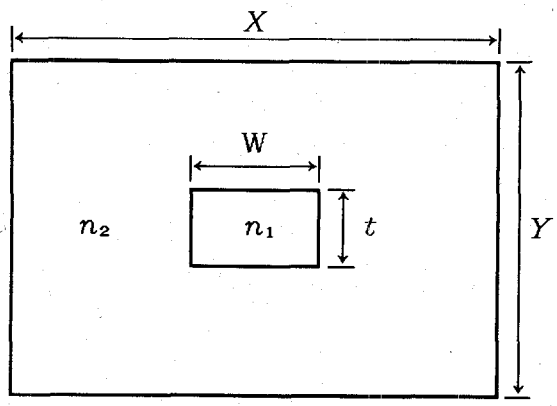

Fig. 4. Dielectric rectangular waveguide.

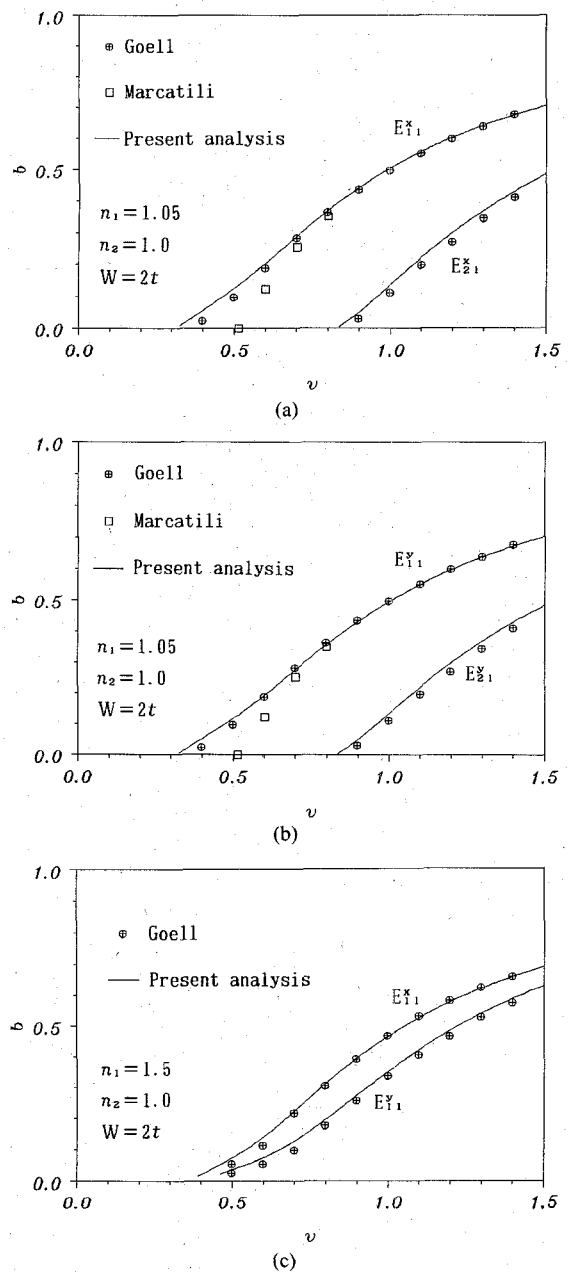

Fig. 5. Propagation characteristics of a dielectric rectangular waveguide. (a) E^x_{11} and E^x_{21} modes (n_1 = 1.05, n_2 = 1.0). (b) E^y_{11} and E^y_{21} modes (n_1 = 1.05, n_2 = 1.0). (c) E^x_{11} and E^y_{11} modes (n_1 = 1.5, n_2 = 1.0).

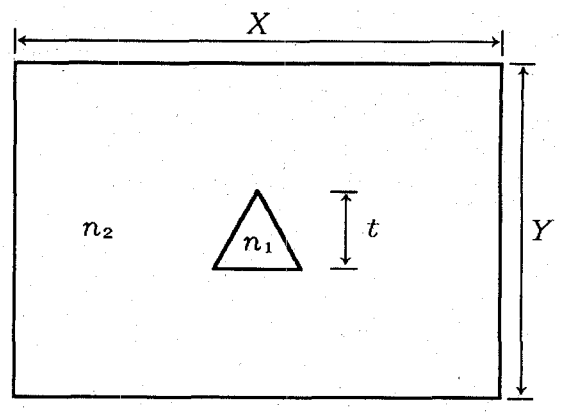

Fig. 6. Equilateral triangular core waveguide.

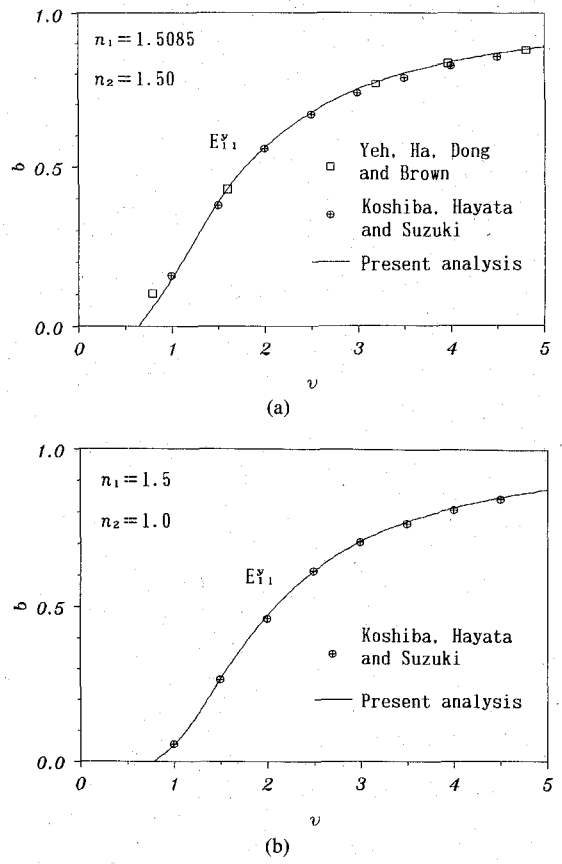

Fig. 7. Propagation characteristics for the E^y_{11} mode of an equilateral triangular core waveguide. (a) n_1 = 1.5085 and n_2 = 1.50. (b) n_1 = 1.5 and n_2 = 1.0.

of elements and the values of X and Y for the influence of the artificial boundaries to be negligible.

V. Conclusion

A simple and efficient finite-element method for the analysis of microwave and optical waveguiding problems was formulated using three components of the electric or magnetic field. In order to eliminate spurious solutions and to treat arbitrarily shaped waveguides, triangular edge elements were utilized. An eigenvalue equation derived here involves only the edge variables in the transversal

plane and can provide a direct solution for the propagation constant. The application of this approach to microstrip lines and dielectric waveguides was also discussed.

This approach can be applied easily to the waveguides including lossy and/or active media, and to the anisotropic waveguides with reflection symmetry [14], [15].

Appendix

The integrals necessary to construct element matrices are calculated as follows:

$$\left[\iint_e \{U\}\{U\}^T\,dx\,dy\right]_{kl} = A_e\tilde{a}_k\tilde{a}_l + A_e y_c(\tilde{a}_k\tilde{c}_l + \tilde{c}_k\tilde{a}_l) + \frac{1}{12}A_e\tilde{c}_k\tilde{c}_l(y_1^2 + y_2^2 + y_3^2 + 9y_c^2) \quad \text{(A1)}$$

$$\left[\iint_e \{V\}\{V\}^T\,dx\,dy\right]_{kl} = A_e\tilde{b}_k\tilde{b}_l - A_e x_c(\tilde{b}_k\tilde{c}_l + \tilde{c}_k\tilde{b}_l) + \frac{1}{12}A_e\tilde{c}_k\tilde{c}_l(x_1^2 + x_2^2 + x_3^2 + 9x_c^2) \quad \text{(A2)}$$

$$\begin{aligned}\left[\iint_e \{U_y\}\{U_y\}^T\,dx\,dy\right]_{kl} &= \left[\iint_e \{V_x\}\{V_x\}^T\,dx\,dy\right]_{kl} \\ &= -\left[\iint_e \{U_y\}\{V_x\}^T\,dx\,dy\right]_{kl} \\ &= -\left[\iint_e \{V_x\}\{U_y\}^T\,dx\,dy\right]_{kl} \\ &= A_e\tilde{c}_k\tilde{c}_l\end{aligned} \quad \text{(A3)}$$

$$\left[\iint_e \{U\}\{N_x\}^T\,dx\,dy\right]_{kl} = \frac{1}{2}(\tilde{a}_k + \tilde{c}_k y_c)b_l \quad \text{(A4)}$$

$$\left[\iint_e \{V\}\{N_y\}^T\,dx\,dy\right]_{kl} = \frac{1}{2}(\tilde{b}_k - \tilde{c}_k x_c)c_l \quad \text{(A5)}$$

$$\iint_e \{N\}\{N\}^T\,dx\,dy = \frac{A_e}{12}\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix} \quad \text{(A6)}$$

$$\iint_e \{N_x\}\{N_x\}^T\,dx\,dy = \frac{1}{4A_e}\begin{bmatrix}b_1^2 & b_1b_2 & b_1b_3\\ b_1b_2 & b_2^2 & b_2b_3\\ b_1b_3 & b_2b_3 & b_3^2\end{bmatrix} \quad \text{(A7)}$$

$$\iint_e \{N_y\}\{N_y\}^T\,dx\,dy = \frac{1}{4A_e}\begin{bmatrix}c_1^2 & c_1c_2 & c_1c_3\\ c_1c_2 & c_2^2 & c_2c_3\\ c_1c_3 & c_2c_3 & c_3^2\end{bmatrix} \quad \text{(A8)}$$

with

$$x_c = (x_1 + x_2 + x_3)/3 \quad \text{(A9)}$$

$$y_c = (y_1 + y_2 + y_3)/3 \quad \text{(A10)}$$

where $[\cdot]_{kl}(kl = 11, 12, \cdots, 33)$ indicates the (k, l) component of the matrix $[\cdot]$.

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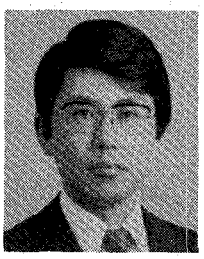

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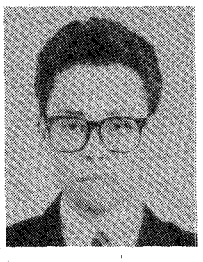

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